**ME EN 541 – HW 2**

Handed out 25 January 2024

Due 1 February 2024

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| **Problem** | **Points** | **Topic** |
| **1** | 5 | Control volume discretization, Practice A |
| **2** | 5 | Control volume discretization, Practice B |
| **3** | 20 | Control volume by hand, constant *k* |
| **4** | 20 | Control volume code, constant *k* |
| **5** | 30 | Control volume code, variable *k* |

1. Derive the discretized form of the convective heat transfer boundary condition for the left boundary of a 1-D steady-state conduction problem using the control volume approach and grid deployment Practice A. Make the standard assumption of a linearized source term, and state your final results in terms of the general FV discretized equation (i.e., explicitly state the coefficients *aw*, *ae*, *aP*, and *b*).

2. Repeat the previous problem for the right boundary and using grid deployment Practice B.

**Problems 3-5:**

Consider one-dimensional conduction in a plane composite wall, as in the following figure. The left side is perfectly insulated. Inside segment A the thermal conductivity is constant and uniform heat generation exists. Inside segment B the thermal conductivity may be a function of *x*, but no generation occurs. The right surface is in contact with a moving fluid. Write a program to calculate the steady-state temperature distribution using the CV method, accounting for non-uniform thermal conductivity and heat generation. Note the following where applicable:

1. Use grid deployment Practice B and equal-sized interior volumes. It is important to have a control surface at the interface between wall segments A and B.
2. Properly evaluate the interface thermal conductivity using the harmonic mean.
3. Solve the linear system using any subroutine of your choosing.

d) Submit a copy of your source code with your HW.

Segment A



Segment B



*T* = 300K

*h* = 1000 W/(m2∙K)

*x*

*x* = *0.03* m

*x* = *0*

*x* = *0.05* m

3. Assume the thermal conductivity of segment B is constant, *kB* = 60 W/(m∙K).

a) Beginning with the governing equation, develop, by hand, the system of equations that corresponds to this problem using the control volume approach with 5 equally-spaced control volumes (i.e., there will be 7 nodes, including the left and right boundaries). Remember to use Practice B. Express your answer in the form **AT** = **b**, where **A** is the matrix of a coefficients, **T** is the temperature vector, and **b** is the vector of *b* values at each node.

b) Solve, by any method, the system of equations. Report *T* for each node.

4. Assume the thermal conductivity of segment B is constant, *kB* = 60 W/(m∙K).

a) Write a code to solve for the temperature at each note using 5 equally-spaced control volumes. Refer to the previous hand-worked problem to verify that your code is set up correctly. Plot the temperature profile and report the temperatures at both boundaries.

b) Repeat using 20 equally-spaced control volumes. Is this result what you expected?

5. Assume the thermal conductivity in segment B varies as *kB*(*x*) = 137*e*25*x* − 2 W/(m∙K).

a) Modify your code to find the temperature for this scenario. Plot the temperature profile and report the temperatures at both boundaries using 5 equal-spaced control volumes.

b) Repeat using 20 equally-spaced control volumes.

c) Plot the predicted maximum temperature in the wall (in Kelvin) as a function of the number of control volumes (i.e., perform a grid-refinement study). What is the “grid-converged” maximum temperature to 3 decimal places (i.e., 0.001), and how many control volumes does it take to obtain this result?